Problem Set 5
Optical Waveguides and Fibers (OWF)

Exercise 1: Qualitative plots of the electromagnetic field components of the $m = 0$ TM mode of a symmetric slab waveguide.

As in the Problem Set 4, consider the slab waveguide depicted in Fig. 1, and assume that the slab is made of silicon, and the substrate and the cladding are made of silica, i.e., $n_2 = n_3 = 1.44$, and $n_1 = 3.48$.

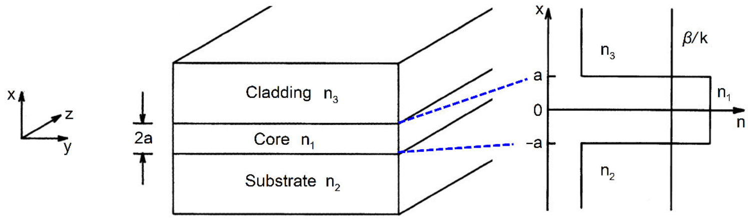

Figure 1: Slab waveguide and coordinate system.

For the TM modes, the mode field $\underline{\mathcal{H}}_y(x)$ is given by

$$\underline{\mathcal{H}}_y(x) = \begin{cases} A\cos(k_{1x}a)\exp\left(-k_{3x}^{(i)}(x-a)\right) & \text{for } a < x \\ A\cos(k_{1x}x) & \text{for } -a \le x \le a \\ A\cos(k_{1x}a)\exp\left(k_{2x}^{(i)}(x+a)\right) & \text{for } x < -a \end{cases} \tag{1}$$

a) Sketch in a *qualitative* way the non-vanishing field components of the fundamental ($m = 0$) TM mode as a function of x.

Solution:

Considering the solution for the TM case that has been derived in the previous problem set, the mode plots are shown in Fig. 2.

b) Explain the behavior of the $\underline{\mathcal{E}}_x(x)$ component at the boundaries by using the continuity of the normal component of the displacement vector $\underline{\mathbf{D}} = \epsilon_0\epsilon_r\underline{\mathbf{E}}$.

Solution:

$$\underline{\mathbf{D}}_{x,1}(x=a) = \underline{\mathbf{D}}_{x,3}(x=a)$$

$$\epsilon_0\epsilon_1\underline{\mathbf{E}}_{x,1}(x=a) = \epsilon_0\epsilon_3\underline{\mathbf{E}}_{x,3}(x=a)$$

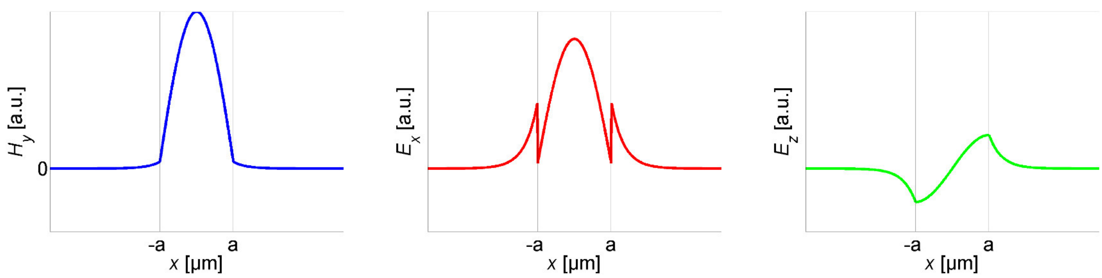

Figure 2: Field plot for the fundamental TM mode of a slab waveguide.

$$\frac{\underline{\mathbf{E}}_{x,1}(x=a)}{\underline{\mathbf{E}}_{x,3}(x=a)} = \frac{\epsilon_3}{\epsilon_1}$$

As we can see from the last line, the $\underline{\mathcal{E}}_x$ component at $x = a$ has to undergo a jump according to the ratio ϵ_3/ϵ_1. The same result holds at $x = -a$.

c) How does the $\underline{\mathcal{E}}_x(x)$ profile of the fundamental TM mode change if you open a narrow silica slot in the center of the slab (see Fig. 3)? Note that narrow slot means $|b| \ll |a|$. A qualitative sketch is sufficient; no quantitative analysis is needed.

Figure 3: Slot waveguide made of silicon and silica.

Solution: If the slot is small enough, then a large electric field propagates within the slot due to the field enhancement calculated in part b). Fig. 4 shows a numerical simulation showing a slot waveguide made of Si and surrounded by SiO_2. Note that the simulated slot waveguide is also confined in the y-direction. However, the $\underline{\mathcal{E}}_x$ component of the slab waveguide shown in Fig. 3 would look qualitatively like the mode profile shown in Fig. 4 (b).

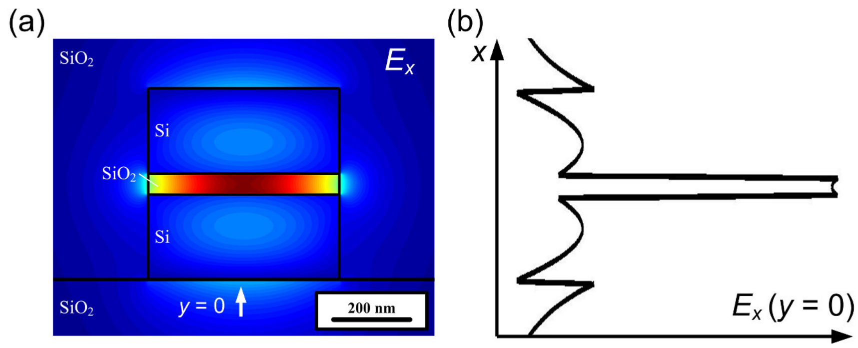

Figure 4: Numerical simulation of a slot waveguide made of silicon and silicon dioxide. Part (a) shows the mode profile of the $\underline{\mathcal{E}}_x$ component and part (b) shows the $\underline{\mathcal{E}}_x$ component at the location $y = 0$, which would correspond qualitatively to the mode profile of a slab waveguide with a narrow slot in the middle, as depicted in Fig. 3.

Questions and Comments:

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